Feature Affinity Assisted Knowledge Distillation and Quantization

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Outline

Knowledge Distillation

Quantization

Experiments

Knowledge Distillation

- Knowledge distillation is the process of transferring knowledge from a large model (teacher) to a smaller one (student) to improve the performance of student model.
- The so-called distillation loss [Hinton et al.'15]

$$L_{\mathrm{KD}}(\theta; x) = CE(\mathrm{softmax}(f_t(x)/\tau), \mathrm{softmax}(f_s(\theta; x)/\tau))$$

$f_s(\theta; x)$ and $f_t(x)$ are logits of the student and teacher resp., $\tau > 0$ is a hyperparameter called temperature,

$$\mathrm{softmax}(\mathbf{z})_i = \frac{\exp(z_i)}{\sum_{j=1}^{k} \exp(z_j)},$$

CE is the cross entropy $CE(p, q) = -\sum_{i=1}^{k} p_i \log q_i$

Knowledge Distillation (Cont'd)

When the (one-hot) label y is available, the regular training loss on the student network is

$$L_{\mathrm{True}}(\theta; x) = \lambda \, CE(\mathrm{softmax}(f_s(\theta; x)), y)$$

The knowledge distillation framework[Hinton et al.'15] requires to solve

$$\min_{\theta} \; \frac{1}{N} \sum_{i=1}^{N} L_{\mathrm{KD}}(\theta; x_i) + \lambda \, L_{\mathrm{True}}(\theta; x_i)$$

where $\lambda > 0$ is a regularization param.

Feature Affinity Matrix

- Consider a (reshaped) feature matrix

$$\mathbf{F} = [\mathbf{f}_1, \ldots, \mathbf{f}_{wh}] \in \mathbb{R}^{c \times wh},$$

the feature affinity matrix $\mathbf{H} \in \mathbb{R}^{wh \times wh}$ is given by the pairwise cosine similarity between two (pixel-wise) feature vectors $\mathbf{f}_i, \mathbf{f}_j \in \mathbb{R}^c$:

$$\mathbf{H}_{ij} := \frac{\langle \mathbf{f}_i, \mathbf{f}_j \rangle}{\|\mathbf{f}_i\| \|\mathbf{f}_j\|} = \cos \theta_{ij}$$

where θ_{ij} is the angle between $\mathbf{f}_i$ and $\mathbf{f}_j$.

Feature Affinity Loss

- At a given pair of layers from student and teacher networks resp., let $\mathbf{F}_s(\theta; x) \in \mathbb{R}^{c_s \times wh}$ and $\mathbf{F}_t(x) \in \mathbb{R}^{c_t \times wh}$ be the features with $c_s < c_t$, and let $\mathbf{H}_s(\theta; x)$, $\mathbf{H}_t(x)$ be the induced feature affinity matrices.
- The feature affinity loss given by l pairs of intermediate features is

$$L_{FA}(\theta; x) = \sum_{j=1}^{l} \frac{1}{w_j^2 h_j^2} \|\mathbf{H}_s^j(\theta; x) - \mathbf{H}_t^j(x)\|_F^2$$

- Feature affinity assisted knowledge distillation gives the sample loss:

$$L(\theta; x) = L_{\mathrm{KD}}(\theta; x) + \lambda_1 \, L_{\mathrm{True}}(\theta; x) + \lambda_2 \, L_{FA}(\theta; x)$$

Drop the second term if labels are unavailable (label-free distillation).

Knowledge Distillation Framework

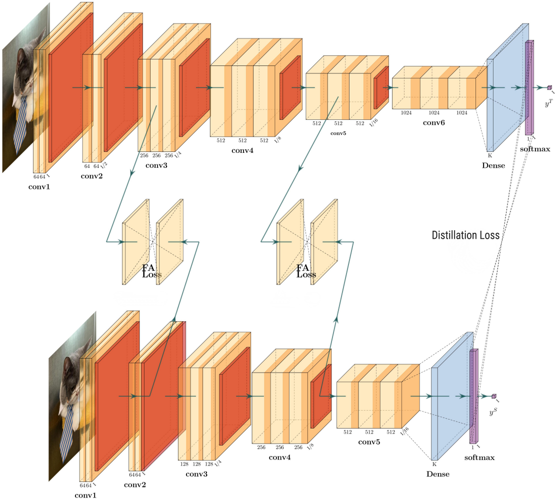


Figure 1: Feature affinity assisted knowledge distillation framework by comparing two sets of feature pairs from the student and teacher.

Existence of Low-Dimensional Feature Embeddings

Denote the cosine similarity by $\|f - g\|_{\cos} := \frac{\langle f, g \rangle}{\|f\| \, \|g\|}$.

Proposition (Johnson-Lindenstrauss-like Lemma)

Given any $\epsilon \in (0,1)$, a feature map $\mathbf{F} = [\mathbf{f_1}, \ldots, \mathbf{f}_n] \in \mathbb{R}^{d \times n}$, with $k = O(\epsilon^{-2} \log n)$, there exists a linear map $T : \mathbb{R}^d \to \mathbb{R}^k$, such that

$$(1-\epsilon)\|\mathbf{f}_i - \mathbf{f}_j\|_{\cos} \leq \|T(\mathbf{f}_i) - T(\mathbf{f}_j)\|_{\cos} \leq (1+\epsilon)\|\mathbf{f}_i - \mathbf{f}_j\|_{\cos}, \ \forall i, j$$

Fast Feature Affinity Loss

- Note that

$$L_{FA}(\theta) := \mathbb{E}_{\mathbf{z}\sim\mathcal{N}(\mathbf{0},\mathbf{I})} \|(\mathbf{H}_s(\theta; x) - \mathbf{H}_t(x))z\|^2$$

- Consider the fast FA loss with k ensembles ($k \ll wh$):

$$L_{fFA,k}(\theta) = \frac{1}{k}\sum_{i=1}^{k} \|(\mathbf{H}_s(\theta; x) - \mathbf{H}_t(x))z_i\|^2$$

where $z_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I}) \in \mathbb{R}^{wh}$

- Concentration inequality:

$$\mathbb{P}\big(|L_{fFA,k}(\theta) - L_{FA}(\theta)| > \epsilon\big) \leq \frac{C}{\epsilon^2 k},$$

where $C = O(L_{FA}(\theta)^4)$.

Outline

Floating Point Representation

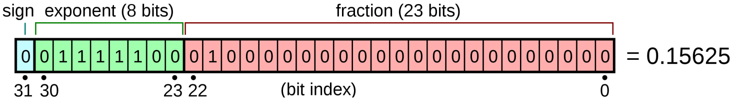
There are 3 elements in a floating point (FP) representation.

- sign
- exponent
- mantissa/fraction

Take FP32 (32-bit) as an example:

$$(-1)^{b_{31}} \cdot 2^{(b_{30}b_{29}\ldots b_{23})_2 - 127} \cdot \left(1 + \frac{b_{22}}{2} + \frac{b_{21}}{2^2} + \cdots + \frac{b_0}{2^{23}}\right)$$

with each $b_i \in \{0, 1\}$.



Integer Representation

For 8-bit integer (INT8):

- signed integer: $-127, -126 \ldots, 127$

$$(-1)^{b_7} \cdot (b_6 b_5 \ldots b_0)_2$$

- unsigned integer: $0, 1, \ldots, 255$

$$(b_7 b_6 \ldots b_0)_2$$

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INT is more efficient than FP in terms of speed, but lacks of precision. Instead consider the scaled INT:

$$\delta \cdot (-1)^{b_7} \cdot (b_6 b_5 \ldots b_0)_2 \quad \text{or} \quad \delta \cdot (b_7 b_6 \ldots b_0)_2$$

allowing some multiplicative FP scalar $\delta > 0$.

INT Quantization

- Learn (scaled) low-bit INT representation (e.g., INT8) for the **weights** and **activation functions** of neural networks. (both the FP scalars and integers)

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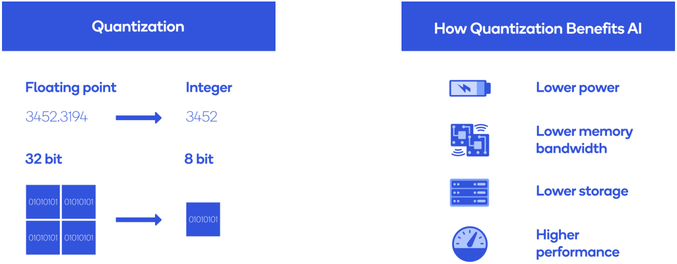
- Learn (scaled) low-bit INT representation (e.g., INT8) for the **weights** and **activation functions** of neural networks. (both the FP scalars and integers)
- In inference phase, accelerate the forward propagation through linear layers:

$$\begin{aligned} W * A &= (\delta \cdot W^{\text{int}}) * (\alpha \cdot A^{\text{int}}) \\ &= (\delta \cdot \alpha) \cdot (W^{\text{int}} * A^{\text{int}}) \end{aligned}$$

  where W and A are quantized weights and activations, resp.
- The FP scalars $\delta, \alpha > 0$ are shared by the whole linear layer and activation layer, resp. (so-called layer-wise quantization).

- Empirically see an up to 16× increase in energy efficiency and a 4× memory savings by going from FP32 to INT8 quantization.



Hardware Implementation

World's first on-device demonstration of Stable Diffusion on an Android phone

Qualcomm AI Research deploys a popular 1B+ parameter foundation model on an edge device through full-stack AI optimization

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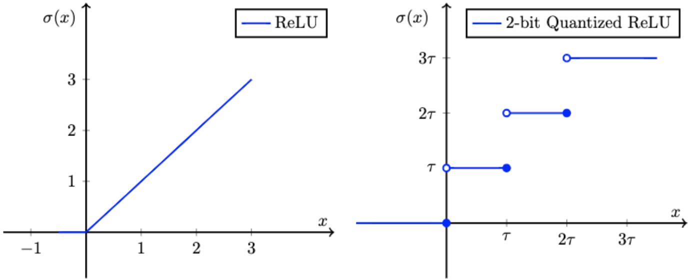
Figure 2: Running INT8 Stable Diffusion model (1B+ params) on Android phones powered by Snapdragon mobile platform takes comparable inference time to that of FP32 model on cloud.

Computational Challenges for Quantization

Solve an optimization problem with

- highly non-convex objective of high dimension
- discrete constraint (quantized weights)
- piecewise constant objective with inapplicable gradient a.e. zero (quantized activations)



Goal: design simple algorithm that

- search along non-gradient based descent direction.
- compute projection efficiently (weight quantization).
- effectively avoid bad local minima.

Training Fully Quantized Neural Networks

$$\min_{\mathbf{w},\boldsymbol{\alpha}} \; f(\mathbf{w},\boldsymbol{\alpha}) := \frac{1}{N}\sum_{i=1}^{N} \ell_i(\mathbf{w},\boldsymbol{\alpha}) \quad \text{subject to} \quad \mathbf{w} \in \mathcal{W}.$$

- $\ell_i(\mathbf{w},\boldsymbol{\alpha}) = \ell(w_L * \sigma(\cdots\sigma(w_1 * x_i, \alpha_1), \dots, \alpha_{L-1}); y_i)$ is the sample loss with or without knowledge distillation.
- $\sigma(x,\alpha)$: unsigned INTq activation function.

$$\sigma(x,\alpha) = \sum_{k=1}^{2^q-2} k\alpha \cdot 1_{\{(k-1)\alpha < x \leq k\alpha\}} + (2^q-1)\alpha \cdot 1_{\{x > (2^q-2)\alpha\}}$$

- $\mathcal{W} = \mathbb{R}_+ \times \{\pm 1\}^n$ for INT1 (a single sign bit), and $\mathcal{W} = \mathbb{R}_+ \times \{0, \pm 1, \dots, \pm(2^{b-1}-1)\}^n$ for signed INTb, $b \geq 2$.

Weight Quantization

Given weights $\mathbf{w}^{(l)} \in \mathbb{R}^n$ (FP32) at Layer l, obtain the INTb quantization by solving

$$\min_{\delta,\mathbf{q}} \|\delta^{(l)} \cdot \mathbf{q}^{(l)} - \mathbf{w}^{(l)}\|^2$$
$$\text{s.t. } \delta^{(l)} > 0,\ \mathbf{q}^{(l)} \in \{0, \pm 1, \ldots, \pm(2^{b-1} - 1)\}^n.$$

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Solve by alternating minimization for $b \geq 2$.

- For $b = 1$, $\mathbf{q}^{(l)} \in \{\pm 1\}^n$, it has closed-form solution [Rastegari et al.'16].

Overcoming Vanished Gradient

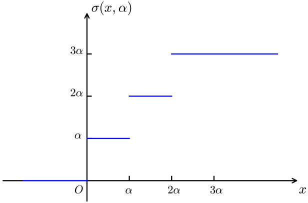
In chain rule, replace $\frac{\partial \sigma}{\partial x}$ with the proxy $\frac{\partial \tilde{\sigma}}{\partial x}$ (so-called straight through estimator [Bengio et al.'13; Yin et al.'19]).

$$\frac{\partial \ell_i(\mathbf{w},\boldsymbol{\alpha})}{\partial \mathbf{w}_{L-1}} \approx \sigma(\mathbf{X}_{L-2},\alpha_{L-2}) \circ \frac{\partial \tilde{\sigma}}{\partial x}(\mathbf{X}_{L-1},\alpha_{L-1}) \circ \mathbf{w}_L^\top \circ \nabla \ell(\mathbf{X}_L; u_i)$$

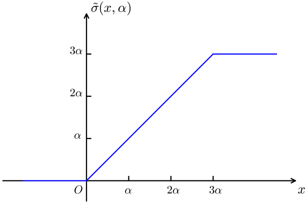
$$\frac{\partial \ell_i(\mathbf{w},\boldsymbol{\alpha})}{\partial \alpha_{L-2}} \approx \frac{\partial \sigma}{\partial \alpha}(\mathbf{X}_{L-2},\alpha_{L-2}) \circ \mathbf{w}_{L-1}^\top \circ \frac{\partial \tilde{\sigma}}{\partial x}(\mathbf{X}_{L-1},\alpha_{L-1}) \circ \mathbf{w}_L^\top \circ \nabla \ell(\mathbf{X}_L; u_i).$$

with $\mathbf{X}_l = \mathbf{w}_l * \sigma(\mathbf{X}_{l-1}, \alpha_{l-1})$ the output from the l-th linear layer.

2-bit quantized ReLU σ



clipped ReLU $\tilde{\sigma}$



- require no extra cost compared with standard gradient computation.

Analysis of Straight Through Estimator

- Given input $\mathbf{x} \in \mathbb{R}^d$ and class label $y \in \{1, \dots, k\}$, consider the two-layer netowrk with output

  $$\mathbf{o}(\mathbf{x}; \mathbf{W}) = \mathbf{V}\sigma(\mathbf{W}\mathbf{x}) \in \mathbb{R}^k$$

  with weights $\mathbf{V} \in \mathbb{R}^{k \times m}$ in the second layer fixed and known. σ is general b-bit activation function:

  $$\sigma(x) = \begin{cases} 0 & \text{if} \quad x \leq 0, \\ \text{ceil}(x) & \text{if} \quad 0 < x < 2^b - 1, \\ 2^b - 1 & \text{if} \quad x \geq 2^b - 1. \end{cases}$$

- $\arg\max_{1 \leq i \leq k} o(x; W)_i$ is the predicted class for x.

- multi-class hinge loss:

$$\ell(\mathbf{W};\mathbf{x},y) = \max\left\{0,\ 1-\left(o(\mathbf{x};\mathbf{W})_y - \max_{i\neq y} o(\mathbf{x};\mathbf{W})_i\right)\right\}$$

- solve the population risk minimization

$$\min_{\mathbf{W}\in\mathbb{R}^{m\times d}}\ f(\mathbf{W}) := \mathbb{E}_{\{\mathbf{x},y\}\sim\mathcal{D}}\left[\ell\left(\mathbf{W};\mathbf{x},y\right)\right],$$

- chain rule to compute partial gradient w.r.t. the j-th row $\mathbf{w}_j^\top$ of $\mathbf{W}$:

$$\begin{aligned}\nabla_{\mathbf{w}_j}\ell(\mathbf{W};\mathbf{x},y) &= (v_{\xi,j}-v_{y,j})\,1_{\{\ell(\mathbf{W};\{\mathbf{x},y\})>0\}}(\mathbf{x})\,\sigma'(\mathbf{w}_j^\top\mathbf{x})\mathbf{x}\\ &= \mathbf{0},\ \text{a.e.}\end{aligned}$$

where $\xi = \text{argmax}_{i\neq y}\, o(x;W)_i$.

Convergence Result

- use (partial) coarse gradient by replacing σ' with μ'

$$\tilde{\nabla}^{\mu}_{\mathbf{w}_j}\ell(\mathbf{W};\mathbf{x},y) := (v_{\xi,j} - v_{y,j})\, 1_{\{\ell(\mathbf{W};\{\mathbf{x},y\})>0\}}(\mathbf{x})\, \mu'(\mathbf{w}_j^\top \mathbf{x})\mathbf{x}.$$

- train the network by coarse gradient algorithm:

$$\mathbf{W}^{t+1} = \mathbf{W}^t - \eta\, \mathbb{E}_{\{\mathbf{x},y\}\sim\mathcal{D}} \tilde{\nabla}^{\mu}\ell(\mathbf{W}^t;\mathbf{x},y)$$

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Theorem (Long, Yin, Xin'21)

Suppose the data from different classes are located in orthogonal subspaces of $\mathbb{R}^d$. *Choose surrogate function* $\mu : \mathbb{R} \to \mathbb{R}$ *satisfying*

1. $\mu(x) = 0$ *for* $x \leq 0$.
2. $\mu'(x) \in [\delta, \tilde{\delta}]$ *for* $x > 0$ *with constants* $0 < \delta < \tilde{\delta} < \infty$.

Then $\lim_{t\to\infty} f(\mathbf{W}^t) = 0$, *leading to perfect classification.*

Full Quantization Algorithm

Algorithm 1 One iteration of Blended Coarse Gradient Descent

Input: mini-batch empirical loss function $f_t(\mathbf{w}, \boldsymbol{\alpha})$, blending parameter $\rho = 10^{-5}$, learning rate $\eta_{\mathbf{w}}^t$ for the weights $\mathbf{w}$, learning rate $\eta_{\boldsymbol{\alpha}}^t$ for the resolutions $\boldsymbol{\alpha}$ (one component per activation layer).

Do:

Evaluate the mini-batch coarse gradient $(\tilde{\nabla}_{\mathbf{w}} f_t, \tilde{\nabla}_{\boldsymbol{\alpha}} f_t)$ at $(\mathbf{w}_Q^t, \boldsymbol{\alpha}^t)$.

$\mathbf{w}^{t+1} = (1-\rho)\mathbf{w}^t + \rho \mathbf{w}_Q^t - \eta_{\mathbf{w}}^t \tilde{\nabla}_{\mathbf{w}} f_t(\mathbf{w}_Q^t, \boldsymbol{\alpha}^t)$ // blended gradient update for weights

$\boldsymbol{\alpha}^{t+1} = \boldsymbol{\alpha}^t - \eta_{\boldsymbol{\alpha}}^t \tilde{\nabla}_{\boldsymbol{\alpha}} f_t(\mathbf{w}_Q^t, \boldsymbol{\alpha}^t)$ // $\eta_{\boldsymbol{\alpha}}^t = 0.01 \cdot \eta_{\mathbf{w}}^t$

$\mathbf{w}_Q^{t+1} = \mathrm{proj}_{\mathcal{W}}(\mathbf{w}^{t+1})$ // quantize the weights

Remark: $\{\mathbf{w}^t\}$ is a sequence of FP-precision auxiliary parameters.

Outline

Experiments

Bitwidth	1W	2W	4W
Cifar-10			
ResNet20 (FP): 92.21 %, Teacher ResNet110			
label-free	89.88%	91.23%	92.19%
with supervision	90.56%	91.65%	92.43%
Cifar-100			
ResNet56 (FP): 72.96%, Teacher ResNet164			
label-free	72.78%	74.35%	74.90%
with supervision	73.35%	74.40%	75.31%
Tiny ImageNet			
ResNet18 (FP): 64.23%, Teacher ResNet34			
label-free FAQD	64.37%	65.05%	65.40%
FAQD with Supervision	65.13%	65.67%	65.92%

Table 1: Wight Quantization (1-bit, 2-bit, or 4-bit) with Feature Affinity Assisted Knowledge Distillation

CIFAR-10		
Pretrained ResNet20: 32W1A-91.89%, 32W4A-92.01%		
Model	1W1A	4W4A
ResNet20	89.70%	92.53%
CIFAR-100		
Pretrained ResNet56: 32W1A-70.96%, 32W4A-71.42%		
Model	1W1A	4W4A
ResNet56	68.18	73.53%
Tiny ImageNet		
Pretrained ResNet18: 32W1A-63.82%, 32W4A-64.15%		
Model	1W1A	4W4A
ResNet18	65.01	65.55%

Table 2: Full quantization on CIFAR-10, CIFAR-100 and Tiny ImageNet, with teacher networks.

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Thank you for your attention!